GUIDOL, Gauhati University
Assignment Based Examination 2021

MSc in Mathematics 3rd Semester

M301: Computer Programming in C

Total Mark: 50

1. Answer any five from the following: 6 x 5 = 30

(a) What do you understand by Data Type? How can you relate variable with data type? Explain with the help of an example.

(b) What do you understand by Operator Precedence and Associativity? Arrange the following operators according to their precedence (low to high).

- * / % ( ) +

Also write down the value 5 / 2 + 10 – 15 * 2.

(c) What is loop? Discuss the three different loop structures available in C.

(d) Write a computer program in C to check an input number is divisible by 11 or not?

(e) Write a computer program in C to display the following pattern:

```
********
******
****
**
*
```

(f) What is Algorithm? Write down the algorithm to check whether an input number is prime or not.

(g) Discuss and implement the Linear Search Algorithm.

2. Write short notes on any four from the following: 5 x 4 = 20

(a) Compiler.

(b) User Defined Data Type.

(c) Header File.

(d) Branching.

(e) **break** and **continue** statements.

(f) Binary Search Algorithm.

Assignment based examination questions
for
M. A. / M. Sc. 3rd Semester in Mathematics under GUIDOL

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Paper:M302 (Number Theory)

Q. 1. Given integers $a$and b, not both of which are zero, show that there exist integers x and y such that $\gcd(a, b) = ax + by$.
Also, show that the set $\{ax + by : x, y \text{ are integers}\}$ is precisely the set of all multiples of $\gcd(a, b)$.
Examine whether $\gcd(2a - 3b, 4a - 5b)$ divides b or not. 8+4+3=15

or

Describe Euclidean Algorithm. Apply this algorithm to calculate gcd(3054, 12378),and express it in the form $ax + by$ in two different ways. 7+3+5=15

Q. 2. For arbitrary integers $a$and b, show that $a \equiv b (mod\ n) \Leftrightarrow a$ and b leave the same non-negative remainder when divided by n. Also, show that the linear congruence $ax \equiv b(mod\ n)$ has a solution if and only if $\gcd(a, n)$divides$b$.
If $ca \equiv cb(mod\ n)$, then show that$a \equiv b(mod\ n/d)$, where $d = \gcd(a, n)$.6+5+4=15

or

State and prove Chinese Remainder Theorem.
Hence, solve the linear congruence $17x \equiv 9(mod\ 276)$ 1+7+7=15

Q. 3. Answer any two of the following: 10x2=20

a) Define quadratic residue of an odd prime p and discuss Euler's Criterion.

b) Find a primitive root of 13. Construct a table of indices with respect to that primitive root of 13. Hence show that the congruence $x^3 \equiv 4(mod\ 13)$ has no solution. Does the congruence $x^3 \equiv 5(mod\ 13)$ have a solution? If yes, find the solution.

c) Show that the functions τ and σ are multiplicative functions. Is the Möbius function μ also multiplicative? Justify your answer.

d) Show that the odd prime number p can be written as a sum of two squares if and only if p is of the form $4n + 1$.

Paper:M303 (Continuum Mechanics)

Q. 1. Answer any three of the following: 5x3=15

(i) Determine the principal stress values and principal direction for the stress tensor

$$\sigma_{ij} = \begin{pmatrix} \tau & \tau & \tau \\ \tau & \tau & \tau \\ \tau & \tau & \tau \end{pmatrix}$$

(ii) The state of stress at a given point is defined by

$$\sigma_{ij} = \begin{pmatrix} \sigma & a\sigma & b\sigma \\ a\sigma & \sigma & c\sigma \\ b\sigma & c\sigma & \sigma \end{pmatrix}$$

where a, b, c are constants and σ is some stress value. Determine the constants a, b, c so that the stress vector on the octahedral plane vanishes.

(iii) A displacement field is given by

$x_1 = X_1 - CX_2 + BX_3,\ \ x_2 = CX_1 + X_2 - AX_3, x_3 = -BX_1 + AX_2 + X_3.$

Show that the displacement represents a rigid body rotation only if the constants A, B, C are very small. Determine the rotation vector $\vec{\omega}$ for the infinitesimal rigid body rotation.

(iv) Derive Bernoulli's equation by integrating Euler's equation along a stream line.

Q. 2. Answer any three of the following: 5x3=15

(i) The stress tensor at a point is given by

$$\sigma_{ij} = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$$

Determine principal stresses, stress invariants and maximum value of shearing stress.

(ii) If a, b, c are all of the same sign, show that the Cauchy stress quadratic for a state of stress represented by

$$\Sigma = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$$

is an ellipsoid.

(iii) A displacement field is given by $x_1 = X_1 + AX_2,\ \ x_2 = X_2 + AX_3,\ x_3 = X_3 + AX_1$, where A is a constant. Calculate the Lagrangian and Eulerian linear strain tensor when A is small.

(iv) Determine the material rate of change of the kinetic energy of the continuum which occupies the volume V and give the meaning of the resulting integrals.

Q. 3. Answer any two of the following: 10x2=20

(i) Discuss the rate of deformation and vorticity with their physical interpretation.

(ii) Write short notes on Elastostatic and Elastodynamics problems.

(iii) What is meant by Barotropic flow? Show that for a Barotropic, inviscid fluid with conservative body forces, the rate of change of circulation is zero. Determine the rate of change of circulation for a perfect fluid with negligible body forces.

Paper:M304 (Algebra II)

(Optional)

Q. 1. Define a lattice as a non-empty set having two binary operations on it. Show that a lattice is a poset. Give an example to show that its converse is not true. Establish that a poset L in which every pair of elements has al.u.b. and g.l.b. in L is a lattice.
2+5+2+6=15

or

Prove that every finite cyclic group is isomorphic to a direct sum of primary cyclic groups. Also, prove that a finitely generated abelian group is the direct sum of a finite set of cyclic group. 8+7=15

Q.2. Show that a free group can be constructed on any non-empty arbitrary set. 15

or

Use Sylow's theorem to show that any group G of order pq where p and q are prime such that $p \not\equiv 1(mod\ q)$ and $q \not\equiv 1(mod\ p)$ is abelian. Also, show that there is no simple group of order 40. Can there be a simple group of order 200? Justify your answer.
7+4+1+3=15

Q.3. Establish any four of the following: 5x4=20

(i) Every proper right ideal in a ring is contained in a maximal proper right ideal.
(ii) A module is Noetherian⟺ every submodule is finitely generated.
(iii) The ideal M of the commutative ring R is maximal if and only if R/M is a field.
(iv) The radical of aright Artinian ring is nilpotent.
(v) If R is right Artinian ring, then any right R-module is Noetherianiff it is Artinian.

Paper:M304 (Space Dynamics)

(Optional)

Q. 1. What is two body problem? Obtain Kepler's equation of motion in the form$m = E - e\sin E$ with usual meanings of the symbols. Also, find the relation between the true anomaly ν and the eccentric anomaly E if $e = \sin\psi$. 2+8+5=15

or

Discuss Gauss's method for determination of the elements of orbit. 15

Q. 2. Define three body problem and give an example.
Derive the first and second integrals of the three body problem. 2+1+12=15

or

Discuss the effects of the tangential component of a perturbing force upon the major axis, eccentricity and the line of apsides. 2+13=15

Q. 3. Answer any two of the following: 10x2=20

(i) Discuss the performance of a single stage rocket and equation for the satellite in gravity free space.

(ii) Derive the expression for distance travelled by a single stage rocket during the burning time of fuel in a constant gravitational field.

(iii)Explain the dynamics of a two stage rocket under the gravity free space.

Paper:M305 (Special Theory of Relativity)

(Optional)

Q. 1. State the two postulates of the Special Theory of Relativity. What are the circumstances that prompted Einstein to formulate these two postulates? Derive, on the basis of these two postulates, the Lorentz transformation equations. 1+5+9=15

or

State Lorentz transformation equations for two frames of reference in uniform relative motion and discuss its consequences (i) Lorentz Fitzgerald contraction, (ii) Time dilation and (iii) Relativity of simultaneity. 1+ 5+5+4=15

Q. 2. Derive the relativistic law of addition of velocities and show that the velocity of light is an absolute constant independent of the motion of the reference system. Also, show that no material particle can move faster than the speed of light. 7+4+4=15

or

Prove that the mass of a body increases with its velocity and its energy equal to the product of its mass and the square of the speed of light. Hence establish the relation between momentum and energy. 6+6+3=15

Q. 3. Show that the space-time interval between any two events is an invariant, and hence discuss and physically interpret the three types of intervals (i) null-like, (ii) space-like and (iii) time-like intervals.
5+5+5+5=20

or

a) Express Lorentz transformations of space and time in four vector form. 4
b) Define electromagnetic field tensor and derive Maxwell's electromagnetic field equations in tensor form. Hence show that the Maxwell equations are invariant under Lorentz transformation. 5+10+1=16

Paper:M305 (Mathematical Logic)

(Optional)

Q. 1. Answer a) and b), or c) and d):

a) Define the following terms: 2x5=10

(i) Statement form

(ii) Invalid argument form

(iii) A proof in L

(iv) A Theorem in L

(v) A Deduction in L

b) Prove that the pairs $\{-,\wedge\}$, $\{-,\vee\}$ and $\{-,\rightarrow\}$ are adequate sets of connections. 5

c) Translate into symbols the following statements: 2x5=10

(i) The sum of two numbers is even if and only if either both numbers are even or both numbers are odd.

(ii) If y is an integer then z is not real, provided that x is a rational number.

(iii) If demand has remained constant and prices have been increased, then turnover must have decreased.

(iv) Not every function has a derivative.

(v) Every number either is negative or has a square root.

d) Prove that $L$is consistent, but far from being complete. 5

Q. 2. Answer any three of the following: 5x3=15

(i) Prove that if L^* is a consistent extension of L, then there is a valuation in which each theorem of L^* takes value T.

(ii) If **A** is a wf of L and **A** is a tautology, then prove that **A** is a theorem of L.

(iii) State and prove the soundness theorem for K.

(iv) Define first order language of $\mathscr{L}$. Give an example of a first order language appropriate for statements about groups.

Q. 3. Answer any two of the following: 10x2=20

(i) Let **A**be the wf of $((\sim p_1 \to p_2) \to (p_1 \to \sim p_2))$. Show that L^+, obtained by including this **A**as anew axiom, has a larger set of theorems than L.
Is L^+ a consistent extension of L? Justify your answer.

(ii) Show that all instances of axiom schemes (K_4), (K_5) and (K_6) are logically valid.

(iii)Write a note on First order systems with equality and the theory of groups in connection with Mathematical system.